Exercise 2

(a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

$$2x^2 + x + xy = 1$$

Solution

Part (a)

Differentiate both sides with respect to x.

$$\frac{d}{dx}(2x^2 + x + xy) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(2x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(xy) = 0$$

$$2\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(xy) = 0$$

$$2(2x) + (1) + \left[\frac{d}{dx}(x)\right] y + x\left[\frac{d}{dx}(y)\right] = 0$$

$$4x + 1 + (1)y + x(y') = 0$$

Solve for y'.

$$y' = -\frac{4x + 1 + y}{x}$$

Part (b)

Solve for y first.

$$xy = 1 - 2x^2 - x$$

$$y = \frac{1 - 2x^2 - x}{x} = x^{-1} - 2x - 1$$

Then take the derivative.

$$\begin{aligned} y' &= \frac{d}{dx}(x^{-1} - 2x - 1) \\ &= -x^{-2} - 2 \end{aligned}$$

Plug the formula for y into the result of part (a) to see if the same answer is obtained.

$$y' = -\frac{4x + 1 + (x^{-1} - 2x - 1)}{x} = -\frac{2x + x^{-1}}{x} = -2 - x^{-2}$$